Corrections to the Bekenstein-Hawking entropy and the Hawking radiation spectrum

July 28, 2011

Brian Kong [1], Youngsub Yoon [2]

[1] *Milton Academy, Milton, MA 02186, USA*
[2] *Harvard University, Cambridge, MA 02138, USA*

Abstract

Firstly, we propose a new area spectrum based on the Einstein-Kaufman pseudo tensor by pointing out an error in the previous derivation of the area spectrum based on Ashtekar's variable; instead of the norm of Ashtekar's gravitational electric field, we show that the norm of our "new" gravitational electric field based on this pseudo tensor gives the correct area spectrum. Remarkably, by using this new area spectrum, we "almost correctly" predict the Bekenstein-Hawking area entropy relation without adjusting Immirzi parameter; we get $0.997\cdots$ for a formula which, when naively calculated, should come out to be 1 if the black hole entropy is $A/4$. We conjecture that this difference 0.003 is due to the extra dimensions which modify the area spectrum; this difference gives a very strong constraint on the size of the extra dimensions which should be understood by future research.

Secondly, we show that the degeneracy of black hole state should be $\sqrt{A}(e^{A/4}-1)$ rather than simply $e^{A/4}$, if the Hawking radiation agrees with the Planck law. By calculating this degeneracy numerically using our new area spectrum, we show that this is the case except for the case when A is small. Also, by the same numerical calculations, we show that the area spectrum predicted by Ashtekar's variable theory doesn't satisfy our degeneracy $\sqrt{A}(e^{A/4}-1)$.

As our radiation spectrum deviates from the Planck law for the small frequency of photon, our area spectrum can be either falsified or verified if the Hawking radiation, with its spectrum, is observed at LHC.

1 Introduction

In his later years, Albert Einstein studied the relativistic theory of the non-symmetric field. In this theory, one does not assume that the metrics and the Christoffel symbols are symmetric; they can be non-symmetric. He published his result with Bruria Kaufman in paper [1], and by himself in book [2]. In this paper and the book, he introduced a pseudo tensor U^l_{ik}, which is related to the Christoffel symbol Γ^l_{ik} by a simple arithmetic formula.

The advantage of using this pseudo tensor over the Christoffel symbol is that Ricci tensor is invariant under the transpose of this pseudo tensor, which Einstein and Kaufman called "transposition invariant." In other words, if the Ricci tensor R_{ik} is expressed by the pseudo tensor U^m_{jl}, one can immediately see that the Ricci tensor R_{ki} is obtained by plugging U^m_{lj} into the previous formula. Of course, the quantum version of this transposition invariant should be Hermiticity, and this pseudo tensor has its advantage, because Hermiticity of an observable means that it gives us a real value.

After writing the Einstein-Hilbert action in terms of this pseudo tensor, by an usual procedure of Euler-Lagrangian and Hamiltonian mechanics, we can easily obtain the poisson bracket of the corresponding canonical variable and its conjugate momentum. In quantum version, this poisson bracket becomes a commutator. This commutator enables us to calculate the spectrum of area and volume, which we obtain to be different from that of Ashtekar's variable theory presented in [4, 5, 11]. We obtained a different result because the paper [11] which claimed that the norm of gravitational electric field gives the area spectrum suffers from an error; in this paper, we suggest that the norm of our "new" gravitational electric field gives the correct area spectrum.

After obtaining the new area spectrum, we plug in this result to a variation of the formula which can be found in [7]. We obtain $0.997\cdots$ for the numerical calculation that should, when naively considered, come out to be 1 if the famous Bekenstein-Hawking entropy formula ($S = A/4$ [12, 13]) is satisfied. The fact that this numerical value we have obtained is so close to 1 strongly suggests that the new area spectrum which we will present in this paper is correct. We conjecture that this difference 0.003 is due to extra dimensions which seem to modify the area spectrum. This concludes the first main argument which is presented from chapter 2 to chapter 6.

In the second phase of our paper we present another convincing proof that this area spectrum obtained in chapter 5 is correct. To this end, we consider the Hawking radiation spectrum. It is very well-known that the black holes must radiate photons of which the spectrum is given by the Planck's law [13]. In 1995, Bekenstein and Mukhanov tried to derive this spectrum on the assumption that there is only one unit area, and all the spectrum of the area are integer multiples of this unit area [15]. Unfortunately,

they failed to reproduce the Planck and Hawking's radiation spectrum exactly because of this wrong assumption.

In section 7, we will generalize Bekenstein and Mukhanov's argument in order to apply it to our case in which there exist not a single unit area, but a multiple number of unit areas. Moreover, in that section, we will show that the number of microstates for the black hole with area A should be given by $\sqrt{A}(e^{A/4}-1)$ rather than $e^{A/4}$ in order that the spectrum of Hawking radiation agree with the Planck's black body radiation spectrum.

In section 8, we will use the area spectrum obtained in chapter 5 to numerically prove that the number of microstates corresponding to the black hole area A does indeed agree with our formula $\sqrt{A}(e^{A/4}-1)$ except for the first few area spectrum. Another point that I want to mention is that we do not yet know how exactly the extra dimensions affect each of the area spectrum. This is the reason why we will just treat as if our new area spectrum were exact; Luckily, they still show remarkable agreements with our formula. One may easily guess that this is so because the real area spectrum modified by extra-dimensional effects are not quite different from our "approximate" area spectrum, since the difference between 1 and 0.997 is very small.

In section 9, by using the same method employed in section 8, we show that the area spectrum predicted by Ashtekar's variable theory imply that the number of microstates corresponding to the black hole area A doesn't agree with $\sqrt{A}(e^{A/4}-1)$. Therefore, the area spectrum based on Ashtekar's variable theory is falsified by numerical calculations.

In section 10, we conclude our paper.

2 Einstein's pseudo tensor

In [1, 2], Einstein introduces Ricci tensor as follows.

$$R_{ik} = \Gamma^s_{ik,s} - \Gamma^s_{is,k} - \Gamma^s_{it}\Gamma^t_{sk} + \Gamma^s_{ik}\Gamma^t_{st} \tag{1}$$

To see that the formula (1) is not transposition invariant, we take its transpose which yields the following:

$$R^*_{ki} = \Gamma^s_{ki,s} - \Gamma^s_{si,k} - \Gamma^s_{ti}\Gamma^t_{ks} + \Gamma^s_{ki}\Gamma^t_{ts} \tag{2}$$

By exchanging k and i in the above formula and changing the dummy variables, we get:

$$R^*_{ik} = \Gamma^s_{ik,s} - \Gamma^s_{sk,i} - \Gamma^s_{it}\Gamma^t_{sk} + \Gamma^s_{ik}\Gamma^t_{ts} \tag{3}$$

which is different from (1). Therefore, the Ricci tensor is not transposition invariant, when written in terms of the Christoffel symbols. To make the Ricci tensor transposition invariant, we introduce the pseudo tensor U^l_{ik} as follows [1, 2]:

$$U^l_{ik} = \Gamma^l_{ik} - \Gamma^t_{it}\delta^l_k \tag{4}$$

Here, we observe that the two terms linear in Γ in (1) is expressed in a single term as $U^s_{ik,s}$. Contracting k and l in (4), we get

$$U^t_{it} = -3\Gamma^t_{it} \tag{5}$$

Therefore, we obtain the following expression for Γ in terms of U:

$$\Gamma^l_{ik} = U^l_{ik} - \frac{1}{3}U^t_{it}\delta^l_k \tag{6}$$

Plugging this formula to (1), we get

$$R_{ik} = U^s_{ik,s} - U^s_{it}U^t_{sk} + \frac{1}{3}U^s_{is}U^t_{tk} \tag{7}$$

which is transposition invariant, as advertised. Now, we have a concrete basis to quantize Einstein's pseudo tensor, which is the main topic of the next chapter.

3 Quantization of Einstein's pseudo tensor

We can write the Einstein-Hilbert action as follows

$$S = \int d^4x\sqrt{-g}g^{ik}R_{ik} \tag{8}$$

where R_{ik} is defined in (7). The equation (8) contains terms with time derivatives which give us dynamics. These terms should be interpreted as canonical variables, and their momentum conjugates are $\sqrt{-g}g^{ik} = \frac{\partial\mathcal{L}}{\partial U^0_{ik,0}}$, where $\mathcal{L}$ is the Lagrangian density, and U^0_{ik}s are canonical variables. Therefore, we can write the following formula, as the poisson bracket of a canonical variable and its conjugate momentum gives the value 1 or delta functions.

$$\{\sqrt{-g}g^{ab}(\vec{\tau}), U^0_{cd}(\vec{\tau'})\} = \delta^a_c\delta^b_d\delta^3(\vec{\tau},\vec{\tau}') \tag{9}$$

Now we will claim the following equation:

$$\sqrt{-g}g^{ab} = D^a_i D^{bi} \tag{10}$$

Here, D is our "new" gravitational electric field, of which the norm is the area. We will prove this formula in section 4. Considering the above formula, we obtain the following:

$$\{D^a_i(\vec{\tau})D^{bi}(\vec{\tau}), U^0_{cd}(\vec{\tau'})\} = \delta^a_c\delta^b_d\delta^3(\vec{\tau},\vec{\tau}') \tag{11}$$

Compare this formula with the result of Ashtekar variables stated in (4.25) in page 148 of [3]. That is:

$$\{A_a^i(\vec{\tau}), E_j^b(\vec{\tau'})\} = \delta_a^b \delta_j^i \delta^3(\vec{\tau}, \vec{\tau}') \tag{12}$$

We find that our formula contains two Ds, while Ashtekar variable formulation has one E.

4 Area is the length of our "new" gravitational electric field

In the paper [11], Rovelli asserted that area is given by the length of gravitational electric field. He obtained this result by considering the area element as follows.

$$E^i(x) = E^{ia}(x)\tilde{\epsilon}_{abc} dx^b \wedge dx^c \tag{13}$$

However, this relation is not correct, because $\tilde{\epsilon}_{abc}$ he considered is not a tensor, (i.e. the Levi-Civita tensor) but just a tensor density or equivalently, a number. (i.e. the Levi-Civita symbol: $\tilde{\epsilon}_{123} = 1$) Therefore $E^{ia}(x)$ considered above is not covariant. By using the above formula Rovelli obtained the following formula.

$$gg^{ab} = E_i^a E^{bi} \tag{14}$$

which is clearly different from our formula (10). Now, let me illustrate how we obtained this formula (10). Let's write the area element as follows:

$$E^i(x) = D^{ia}(x)\epsilon_{abc} dx^b \wedge dx^c \tag{15}$$

where ϵ_{abc} is a tensor. (i.e. $\epsilon_{123} = \sqrt{-g}$) Equivalently, if we use vierbein formalism, the area element is simply given by the wedge product between two vierbein field as follows:

$$E(x) = \tilde{\epsilon}_{abc} e^b \wedge e^c \tag{16}$$

where $\tilde{\epsilon}$ is a tensor density defined by $\tilde{\epsilon}_{123} = 1$. Now let's relate the formula (15) and (16). Instead of calculating their norms and equate them, let's just calculate the length squared of each of them and equate them, since if their length squared are the same, their norms are the same. To find the length squared, we will do a dot-product with itself.

First, from (15), we get

$$\begin{aligned} \int < E^i, E^i > &= \int < D^{ia}\epsilon_{abc} dx^b \wedge dx^c, D^{if}\epsilon_{fed} dx^e \wedge dx^d > \\ &= \int D^{ia} D^{if} \epsilon_{abc}\epsilon_{fed}(g^{be}g^{cd} - g^{bd}g^{ce}) d^4x \end{aligned} \tag{17}$$

On the other hand, from (16), we get

$$\int < E^i, E^i >= \int (\tilde{\epsilon}_{abc} e^b \wedge e^c) \wedge (e^a \wedge e^0) = \int \sqrt{-g} d^4x \tag{18}$$

Here, we used the fact that the Hodge dual of $(\tilde{\epsilon}_{abc} e^b \wedge e^c)$ is $(e^a \wedge e^0)$. Now, we can write the above formula slightly differently as follows.

$$\int \sqrt{-g} d^4x = \int \sqrt{-g} g^{af} \epsilon_{abc} \epsilon_{fed} (g^{be} g^{cd} - g^{bd} g^{ce}) d^4x \tag{19}$$

Equating this with (17), we obtain:

$$\sqrt{-g} g^{af} = \sum_i D^{ia} D^{if} \tag{20}$$

which we claimed before in (10).

5 The spectrum of area

The fact that our formula (11) contains two Ds instead of one E as in (12) in the commutator has a far reaching consequence in calculating the spectrum of area.

As explained earlier, according to the loop quantum gravity based on Ashtekar's variable, the spectrum of area is calculated to be the eigenvalues of $\sqrt{E_i^a E^{bi}}$ [3, 5, 11]. In Ashtekar's and Rovelli's theory, as E is the conjugate momentum of A, to calculate the previous formula, we have to replace Es by derivatives with respect to As, the connections. Explicitly, it means the following [3]:

$$E_i^a \Psi_s(A) = \frac{\delta}{\delta A_a^i} \Psi_s(A) \tag{21}$$

Here $\Psi_s(A)$ is the spin network state.

On the other hand, in Einstein's pseudo tensor theory, one has to differentiate the spin network state with respect to the pseudo tensor U instead of the connection A. Before calculating this explicitly, let me mention that A^i is related to $\Gamma_{\mu i}^0$ in a natural way as follows:

$$A_\mu^i dx^\mu = \omega_{\mu i}^0 dx^\mu = \Gamma_{\mu i}^0 dx^\mu \tag{22}$$

Therefore, taking derivatives with respect to A^i is the same as taking derivatives with respect to $\Gamma_{\mu i}^0$. Moreover, if we take the path for the holonomy at equal times which is usual and natural, we get that differentiating the holonomy with respect to $\Gamma_{\mu 0}^0$ is equal to zero. Therefore, when differentiating the holonomy with respect to Γ_{jm}^0 we only need to consider the case when $m \neq 0$. This fact has the following consequences:

$$\frac{\partial \Gamma^0_{jm}}{\partial U^0_{jm}} = 1 - \frac{1}{3}\delta^0_m = 1 \tag{23}$$

As we apply this formula to the chain rule, we can say that differentiating with respect to the connection A^i_a is equivalent to differentiating with respect to the pseudo tensor U^0_{ai} as follows.

$$\frac{\delta}{\delta A^i_a}\Psi_s(A) = \frac{\delta}{\delta \Gamma^0_{ai}}\Psi_s(A) = \frac{\delta}{\delta U^0_{ai}}\Psi_s(A) \tag{24}$$

However, considering (11), in this case we observe that taking the derivative with respect to U brings down two Ds instead of one E as in (21) in the Rovelli's theory. This means the following:

$$\frac{\delta}{\delta U^0_{ab}}\Psi_s(A) = D^a_i D^{bi}\Psi_s(A) \tag{25}$$

Comparing the above formula with (21), we can understand that the eigenvalues of Ds of loop quantum gravity based on Einstein's pseudo tensor is the square root of the eigenvalues of Es (gravitational electric field) in loop quantum gravity based on Ashtekar's variable. Thus, we conclude that the area spectrum obtained by our theory is the square root of those of Ashtekar's variable. Therefore, to obtain the area spectrum predicted by our theory, we first need to look at the general area spectrum predicted by Ashtekar's and Rovelli's theory, because all we need to do is taking the square root of it. To this end, instead of writing down the derivation of the general area spectrum obtained by Ashtekar's variable theory, we simply quote the result. [7, 8]

$$A = 4\pi\gamma \sum_i \sqrt{2j^u_i(j^u_i+1) + 2j^d_i(j^d_i+1) - j^t_i(j^t_i+1)} \tag{26}$$

where γ is the Immirzi parameter and j^u_i, j^d_i, j^t_i are quantum numbers which satisfy the constraints found in [7]; i.e. j^u_i, j^d_i are non-negative half-integers, j^t_i is a non-negative integer, while the sum of these three numbers should be an integer and any of the two numbers in this set of three numbers is bigger than or equal to the other one number. The authors of [7] note that the condition that j^t_i is an integer is "motivated by the ABCK framework where the "classical horizon" is described by U(1) connection."[7, 14] Now, the area spectrum of our theory is the following, as it is the square root of Ashtekar's variable theory:

$$A = 8\pi \sum_i \sqrt{\frac{1}{2}\sqrt{2j^u_i(j^u_i+1) + 2j^d_i(j^d_i+1) - j^t_i(j^t_i+1)}} \tag{27}$$

6 Black hole entropy calculation

This section closely follows [7]. To understand the formula which can check whether the black hole entropy is $A/4$, we reconsider the "simplified area spectrum" as follows. We consider the following number of states $N(A)$.

$$N(A) := \left\{ (j_1, \cdots, j_n) | 0 \neq j_i \in \frac{\mathbb{N}}{2}, \sum_i \sqrt{j_i(j_i+1)} = \frac{A}{8\pi\gamma} \right\} \tag{28}$$

We derive a recursion relation to obtain the value of $N(A)$. When we consider $(j_1, \cdots, j_n) \in N(A - a_{1/2})$ we obtain $(j_1, \cdots, j_n, \frac{1}{2}) \in N(A)$, where $a_{1/2}$ is the minimum area where only one $j = 1/2$ edge contributes to the area eigenvalue. i.e., $a_{1/2} = 8\pi\gamma\sqrt{\frac{1}{2}(\frac{1}{2}+1)} = 4\pi\gamma\sqrt{3}$. Likewise, for any eigenvalue $a_{j_x}(0 < a_{j_x} \leq A)$ of the area operator, we have

$$(j_1, \cdots, j_n) \in N(A - a_{j_x}) \Longrightarrow (j_1, \cdots, j_n, j_x) \in N(A). \tag{29}$$

Then, important point is that if we consider all $0 < a_{j_x} \leq A$ and $(j_1, \cdots, j_n) \in N(A - a_{j_x})$, $(j_1, \cdots, j_n, j_x)$ form the entire set $N(A)$. Thus, we obtain

$$N(A) = \sum_j N(A - 8\pi\gamma\sqrt{j(j+1)}). \tag{30}$$

By plugging $N(A) = \exp(A/4)$, one can determine whether the above formula satisfies Bekenstein-Hawking entropy formula. If the Bekenstein-Hawking entropy is satisfied, the following should be satisfied [16]:

$$1 = \sum_j \exp(-8\pi\gamma\sqrt{j(j+1)}/4) \tag{31}$$

Now, we can simply generalize the above formula to the case of general area spectrum as follows.

$$\begin{aligned} N(A) := \quad & \left\{ (j_1^u, j_1^d, j_1^t, \cdots, j_n^u, j_n^d, j_n^t) | j_i^u, j_i^d \in \frac{\mathbb{N}}{2}, \ \ j_i^t \in \mathbb{N}, \ \ j_i^u + j_i^d + j_i^t \in \mathbb{N}, \ j_i^1 \leqq j_i^2 + j_i^3 \right. \\ & \left. \sum_i \sqrt{\frac{1}{2}\sqrt{2j_i^u(j_i^u+1) + 2j_i^d(j_i^d+1) - j_i^t(j_i^t+1)}} = \frac{A}{8\pi} \right\} \ . \end{aligned} \tag{32}$$

In the paper [7], the authors change the variables j_i^u, j_i^d, j_i^t to integers to calculate easily. We will not show these details here. One may easily refer to this construction in their paper. Then, they consider"the proposal that we should count not only j but also $m = -j, -j+1, \cdots, j-1, j$ freedom based on the ABCK framework."[7, 14] Then, they go on to claim that counting only m related to j^u and j^d "is reasonable from the point

of view of the entanglement entropy [17] [18] ([20]) or the holographic principle [19]"[7]. Thus, we get the following formula which is a generalization of (31).

$$1 = \sum_i \{(2j_i^u + 1) + (2j_i^d + 1)\} \exp(-8\pi\sqrt{\frac{1}{2}\sqrt{2j_i^u(j_i^u + 1) + 2j_i^d(j_i^d + 1) - j_i^t(j_i^t + 1)}}/4) \tag{33}$$

One thing to comment on the above formula is that the authors of [7] incorrectly put $2j_i^u + 1$ instead of $(2j_i^u + 1) + (2j_i^d + 1)$ when j_i^u is equal to j_i^d. By calculating using Mathematica, we obtained $0.997\cdots$ for the right-hand side of (33). The fact that the right-hand side is so close to 1 suggests that the general area spectrum obtained by applying Einstein's pseudo tensor into loop quantum gravity is on the right track. We hope that this very small difference between $0.997\cdots$ and 1 will be understood in terms of the effects of extra dimensions.

7 Derivation of the number of states for a given area

The intensity of light with given frequency ν and with given radiating object's area A in the black body radiation is given by the following formula, the Plank's law.

$$dI = \frac{2h\nu^3}{c^2}\frac{Ad\nu}{e^{h\nu/kT} - 1} \tag{34}$$

Equivalently, the number of photons produced is given by:

$$dn_{photon} = \frac{2\nu^2}{c^2}\frac{Ad\nu}{e^{h\nu/kT} - 1} \tag{35}$$

In the case of the black hole, the black hole radiates certain frequencies of light corresponding to the spectrum of areas. As is the case in [15], it is easy to see that $h\nu/kT$ should correspond to $A_{spec}/4$, where A_{spec} is the area spectrum, since the leading behavior in the denominator should reproduce the black hole entropy. From [13], we also know that $1/kT = 8\pi M$ and $A_{BH} = 16\pi M^2$ where M is the mass of the black hole, and A_{BH} the area of the black hole. Plugging everything into (35), we obtain the following:

$$dn_{photon} = \frac{A_{spec}^2 dA_{spec}}{2048\pi^{9/2}\sqrt{A_{BH}}(e^{A_{spec}/4} - 1)} \tag{36}$$

Considering the fact that the numerator on the above equation is merely a phase space factor and the general argument presented in [15], we can conclude that the denominator should give the number of states for the area A if $A = A_{BH} = A_{spec}$. Therefore, we argue that the number of states for the area between A and $A + dA$ is given by the following formula:

$$dN(A) = \frac{1}{C}\sqrt{A}(e^{A/4} - 1)dA \tag{37}$$

Where C is a constant we failed to calculate analytically, even though it may naively seem that it can be easily derived from (36). We will determine this constant by a numerical method in the next chapter.

8 Another verification of our area spectrum: the number of states

The area spectrum we obtained in chapter 5 is not exact, as there is a difference between 0.997 and 1. So, it may be a wrong way to use our non-exact area spectrum to verify our formula (37). However, our area spectrum is almost correct, since the difference between 0.997 and 1 is very small. So, we may as well use our non-exact area spectrum to verify this formula (37), since there is no way to obtain the exact area spectrum at this point.

To this end, we integrate both hand-sides of (37), then we get the following:

$$C(A) = \frac{I(A)}{N(A)} \tag{38}$$

where $N(A)$ is the number of states between 0 and A (including A), and $I(A)$ is given by:

$$I(A) = \int_0^A \sqrt{A'}(e^{A'/4} - 1)dA' \tag{39}$$

In an ideal case, when our area spectrum totally respects the Planck law, $C(A)$ is a constant that doesn't depend on A.

In addition, to calculate easily, we define a variable x as following:

$$x = 2j_i^u(j_i^u + 1) + 2j_i^d(j_i^d + 1) - j_i^t(j_i^t + 1) \tag{40}$$

Then, we can easily see the following relation:

$$A(x) = 8\pi\sqrt{\frac{1}{2}\sqrt{x}} \tag{41}$$

We will use this notation for the following table. Putting everything together, and using computer, we obtain Table 1.

In the table, we can see that the value of C quickly stabilizes after the first several spectrum and converges to a fixed value 172∼173 for the higher frequencies.

Table 1: C(A), pseudo tensor case

x	A	N(A)	I(A)	C(A)
1	17.8	4	1193	298.2
2	21.1	14	3166	226.1
3	23.4	32	5977	186.8
4	25.1	50	9702	194.0
5	26.6	72	14425	200.3
6	27.8	110	20240	184.0
7	28.9	154	27242	176.9
8	29.9	204	35535	174.2
9	30.8	262	45223	172.6
10	31.6	326	56416	173.1
11	32.4	388	69229	178.4
12	33.1	474	83778	176.7
13	33.7	584	100187	171.6
14	34.4	684	118581	173.4
15	35.0	804	139090	173.0

9 Falsification of Ashtekar-Rovelli model

As mentioned in section 6,"the authors of [7] incorrectly put $2j_i^u + 1$ instead of $(2j_i^u + 1) + (2j_i^d + 1)$ when j_i^u is equal to j_i^d" when calculating the number of states. Fixing this mistake, we obtained $0.85417\cdots$ for the Immirzi parameter. This value of the Immirzi parameter is the one we will use to falsify Ashtekar-Rovelli model.

As in section 8, (39) and (40) are still valid, but (41) is not. (41) should be modified by the following formula:

$$A(x) = 4\pi\gamma\sqrt{x} \tag{42}$$

where γ is $0.85417\cdots$. Putting everything together, we obtain Table 2.

In the table, we can see that C doesn't converge, it keeps growing.

Therefore, we conclude that the area spectrum based on Ashtekar's variable doesn't reproduce the Planck law.

10 Discussion and Conclusions

In the first part of our paper we introduced Einstein's pseudo tensor, and by quantizing it, we applied it to calculating the spectrum of area. We obtained that the spectrum of

Table 2: C(A), Ashtekar variable case

x	A	N(A)	I(A)	C(A)
1	10.7	4	123	30.7
2	15.2	14	543	38.8
3	18.6	32	1520	47.5
4	21.5	66	3481	52.7
5	24.0	88	7091	80.6
6	26.3	206	13351	64.8
7	28.4	250	23733	94.9
8	30.4	544	40364	74.2
9	32.2	810	66260	81.8
10	33.9	1234	105637	85.6

area is the square root of the spectrum of area predicted by Ashtekar's variable theory. By using this result for the area spectrum, we showed that our result "almost correctly" predicts the Bekenstein-Hawking entropy formula. This is very remarkable because no one, in the current framework of loop quantum gravity, has succeeded yet to predict Bekenstein-Hawking entropy formula without adjusting Immirzi parameter. Of course, it's because Immirzi parameter, which is explained in [6], plays an indispensable role in current framework of loop quantum gravity. Nevertheless, a consistent check is lacking, as no one has computed the value of Immirzi parameter by other ways than adjusting it to make the Bekenstein-Hawking entropy formula hold. Therefore, our theory is very advantageous.

We also conjecture that the very small discrepancy between $0.997\cdots$ and 1 is due to the extra dimensions which seem to modify the area spectrum. We hope that this discrepancy will enable us to determine at least the length scale of the compact extra dimensions by future research. For example, one may check whether Arkani-hamed and Dimopoulos's proposal [9] that new dimensions are at a millimeter is consistent with this very small discrepancy. Or, one may also check whether Randall-Sundrum model [10] is consistent with our theory. In any case, we hope that our theory will be a useful tool to probe extra dimensions; our theory gives a very strong constraint on the size of the extra dimensions.

In the second part of our paper, we numerically showed that our area spectrum agrees with the Hawking radiation spectrum, or equivalently the Plank law, which is remarkable, because the area spectrum obtained using Ashtekar's variable doesn't reproduce the Hawking radiation spectrum. It is evident that the loop quantum gravity theory based on Ashtekar's variable and Immirzi parameter should be abandoned, because they

are not related to the area spectrum.

Finally, our new area spectrum gives falsifiable predictions should it be the case that Hawking radiation is observed at LHC; the Hawking radiation spectrum is discrete rather than continuous, because the area spectrum is quantized. As there is a gap between zero and the smallest value of area allowed, there are no photons emitted below a certain frequency which we calculate to be $h\nu/kT \approx 4.44$. We hope that this is verified at LHC.

11 Acknowledgment

BK calculated $N(A)$ in the table 1 and 2 using java. YY did the rest of the work.

References

[1] A. Einstein and B. Kaufman, "A new form of the general relativistic field equations" Annals Math. **62**, 128 (1955).

[2] Albert Einstein "The Meaning of Relativity - Including the Relativistic Theory of the Non-Symmetric Field"

[3] C. Rovelli, "Quantum Gravity" *Cambridge, UK: Univ. Pr. (2004) 455 p*

[4] A. Ashtekar, "New Variables for Classical and Quantum Gravity," Phys. Rev. Lett. **57**, 2244 (1986).

[5] C. Rovelli and L. Smolin, "Discreteness of area and volume in quantum gravity," Nucl. Phys. B **442**, 593 (1995) [Erratum-ibid. B **456**, 753 (1995)] [arXiv:gr-qc/9411005].

[6] C. Rovelli and T. Thiemann, "The Immirzi parameter in quantum general relativity," Phys. Rev. D **57**, 1009 (1998) [arXiv:gr-qc/9705059].

[7] T. Tanaka and T. Tamaki, "Black hole entropy for the general area spectrum," arXiv:0808.4056 [hep-th].

[8] S. Frittelli, L. Lehner and C. Rovelli, "The complete spectrum of the area from recoupling theory in loop quantum gravity," Class. Quant. Grav. **13**, 2921 (1996) [arXiv:gr-qc/9608043].

A. Ashtekar and J. Lewandowski, "Quantum theory of geometry. I: Area operators," Class. Quant. Grav. **14**, A55 (1997) [arXiv:gr-qc/9602046].

[9] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, "The hierarchy problem and new dimensions at a millimeter," Phys. Lett. B **429**, 263 (1998) [arXiv:hep-ph/9803315].

I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, "New dimensions at a millimeter to a Fermi and superstrings at a TeV," Phys. Lett. B **436**, 257 (1998) [arXiv:hep-ph/9804398].

[10] L. Randall and R. Sundrum, "A large mass hierarchy from a small extra dimension," Phys. Rev. Lett. **83**, 3370 (1999) [arXiv:hep-ph/9905221].

L. Randall and R. Sundrum, "An alternative to compactification," Phys. Rev. Lett. **83**, 4690 (1999) [arXiv:hep-th/9906064].

[11] C. Rovelli, "Area is the length of Ashtekar's triad field," Phys. Rev. D **47**, 1703 (1993).

[12] J. D. Bekenstein, “Black holes and entropy,” Phys. Rev. D **7**, 2333 (1973).

[13] S. W. Hawking, “Particle Creation By Black Holes,” Commun. Math. Phys. **43**, 199 (1975) [Erratum-ibid. **46**, 206 (1976)].

[14] A. Ashtekar, J. Baez, A. Corichi and K. Krasnov, Phys. Rev. Lett. **80**, 904 (1998) [arXiv:gr-qc/9710007].

A. Ashtekar, A. Corichi and K. Krasnov, Adv. Theor. Math. Phys. **3**, 419 (2000) [arXiv:gr-qc/9905089].

A. Ashtekar, J. C. Baez and K. Krasnov, Adv. Theor. Math. Phys. **4**, 1 (2000) [arXiv:gr-qc/0005126].

[15] J. D. Bekenstein and V. F. Mukhanov, “Spectroscopy of the quantum black hole,” Phys. Lett. B **360**, 7 (1995) [arXiv:gr-qc/9505012].

[16] K. A. Meissner, “Black hole entropy in loop quantum gravity,” Class. Quant. Grav. **21**, 5245 (2004) [arXiv:gr-qc/0407052].

[17] For review, see, e.g., M. A. Nielsen and I. L. Chuang, Modern Quantum Computation and Quantum Information (Cambridge University Press, Cambridge, 2000).

[18] L. Bombelli, R. K. Koul, J. H. Lee and R. D. Sorkin, “A Quantum Source of Entropy for Black Holes,” Phys. Rev. D **34**, 373 (1986).

[19] See, e.g., ’t Hooft, hep-th/0003004.

[20] D. R. Terno, “From qubits to black holes: Entropy, entanglement and all that,” Int. J. Mod. Phys. D **14**, 2307 (2005) [arXiv:gr-qc/0505068].

E. R. Livine and D. R. Terno, “Quantum black holes: Entropy and entanglement on the horizon,” Nucl. Phys. B **741**, 131 (2006) [arXiv:gr-qc/0508085].

W. Donnelly, “Entanglement Entropy in Loop Quantum Gravity,” Phys. Rev. D **77**, 104006 (2008) [arXiv:0802.0880 [gr-qc]].